# PROJECT REPORT

# TENSEGRITY BASED MORPHING STRUCTURES

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**Abstract**

In the duration of the REU academic year program, the scope of this project involved the exploration of applications of tensegrity-based morphing structures. Tensegrity systems are easily manipulated, robust, and stable. These properties allow for engineers to create cost-effective and unique solutions to existing problems. This project aims to use the properties of tensegrity and apply them to flight, namely kite flying, in order to solve some frequently-encountered problems associated with flying kites. As it is now, flying box kites, is an enjoyable activity, but it is impossible to separate the act of flying box kites from the task of assembling the kite, initially keeping the kite aloft, and doing either without the assistance of another individual. These tasks are not too difficult, they are not so annoying that they deter kite enthusiasts from flying their box kites, but they do detract from the overall experience, and the application of tensegrity could potentially solve the problem. A tensegrity configuration would allow the collapse of the kite in such a way that it can be deployed back to its operating shape in a matter of seconds, and also compressed into a compact form conducive to launching from a device which employs a propulsion mechanism. Such a development would eliminate the need for assistance from another individual.

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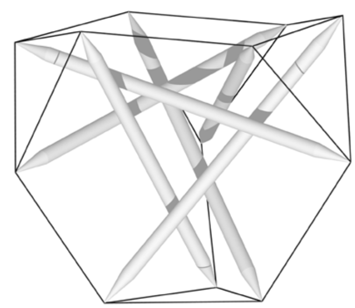
## INTRODUCTION

### 1.1. Tensegrity

Tensegrity structures consist of strings in tension and bars in compression. The word "tensegrity" comes from the combination of the words "tension" and "integrity". Initially coined by R. Buckminister Fuller, the word adequately describes the concept. Tensegrity is characterized by an arrangement of rigid members or characters and tensile members in such a way that the entire configuration is kept stable purely by internal forces. Immediately, one can notice the aesthetic appeals of tensegrity, as examples can be found in art and architecture, but tensegrity allows for the construction of lightweight, robust, and morphable structures that engineers and scientists are only beginning to apply to practical use. Tensegrity in structural modeling has resulted in significant weight and economic savings, up to 60 percent. Tensegrity also opens the door to morphable, collapsible, and deployable structures, thus increasing their versatility, portability, and utility. Engineers often combine the concept of tensegrity with active controls. Actuators can drastically change the shape of a tensegrity system and engineers can benefit from this offered flexibility and deformation.



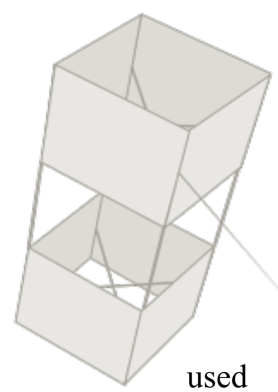
Tensegrity is the relationship between tensile and compressive forces within a structure that has members that distinctly are under one type of force or the other. Tensegrity systems exhibit some the forces ordinary structures undergo, but far more explicitly by isolating the different forces at work. Fuller asserts that tensegrity systems are that which meet the following conditions: First, tensegrity systems are comprised of solid members supported by tension forces and do not support one another. These forces can be supplied by tensile members, or also can be distributed among the rigid members like in a geodesic dome. The rigid members can be connected, but only in such a way that the compressive forces are not transferred to one another. Second, the structures must not depend on weight, that is, the systems must be able to support itself even in the absence of gravity. All tensegrity systems are created on Earth and are therefore subject to the force of gravity, but true tensegrity systems are stable even without the weight of any of the members acting to support the system. Finally, tensegrity systems are arranged in such a way that all members are subject to deformations of one, meaning that a change of one member is reflected throughout the entire structure.

> Snelson: … [It] describes a closed structural system composed of a set of three or more elongate compression struts within a network of tension tendons, the combined parts mutually supportive in such a way that the struts do not touch one another, but press outwardly against nodal points in the tension network to form a firm, triangulated, prestressed, tension and compression unit.

In engineering, it is well-known that equilibrium comes from a balance of forces in three dimensions. Tensegrity systems would never be stable if they did not exhibit this balance. This is achieved when the reactive compressive forces act "outward" along the members and they "cancel out" the "inward" acting forces within the tensile members. The same could be said about the moments all of the forces generate on the system. Forces that cause a moment in one direction will always additively cancel out forces that tend to cause a moment in the other. Such is the case with this project, twisting moments are apparent within the structure of the models, but internally, these imbalances are resolved and the models are all stable. Tensegrity systems are often complex because both the forces and moments need to balance out. So even though a member's forces are correctly balanced out with the forces of one or two other members, typically there is a resultant moment which needs to be counter-acted with an additional member, which leads to the need of an additional member or members to act against the force the new member introduces, and so forth.

Usually, when a tensegrity system is subjected to external forces, it can deform to accommodate them. Although there are two-dimensional tensegrity systems, typically tensegrity systems occur in space and in order for the system to be stable, movement in the six degrees of freedom (translation and torque in all three spatial dimensions) must be controlled.

## 1.2. Box Kites

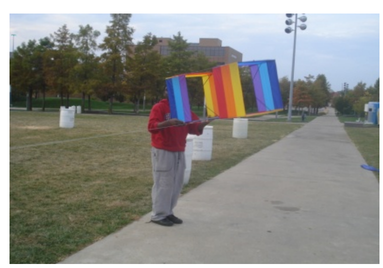


The conception of box kites was instrumental to unlocking the secret to manned flight. Invented by Lawrence Hargrave and later adapted by the Wright brothers, box kites were incorporated into the first aircraft design. More than two centuries later, box kites are still used in recreational kite-flying today. The box kite features an arrangement of rods into a boxed shape, held together by cross members at the ends. Sails are affixed to the configuration and the arrangement, along with the low mass-to-volume ratio (density) allows for sustained flight, provided the right conditions. Internally, box kites are a self-supporting structure. Their shape can be held standing on end or lying flat on the ground. When flying in the air, the box shape is held. The box kite's appeal comes from the fact that the structure is simple and easily duplicated, lightweight, and its ability to be modified, for example, the addition of wings or a repeated array of boxes.

Box kites were particularly appealing because of their ability to collapse into a tube or package. A simplistic form of tensegrity is applied to the traditional box kite. Rods through the center are compressed while the tensile forces of the sails hold its shape. Negative aspects of box kites include setup time and the need of two people to launch. It takes time to assemble a kite after removing it from a container. Compressing the rods to hold the shape of the sails can be difficult. It requires effort to force them into place and sensitivity to not break the rods. Without a second person, lifting the kite off the ground would be near impossible unless the winds are extraordinarily strong. Box kites are fantastic toys and works of art; they are physical wonders and a scientific stepping stone. However, they are by no means perfect creations.

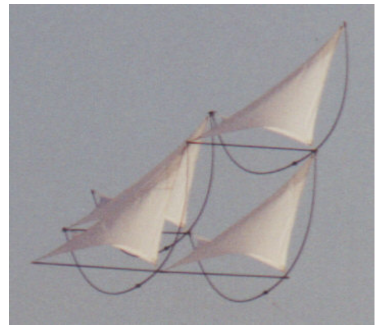
# 2. GOAL AND PROJECT OBJECTIVES

## 2.1. Opportunity



Anyone familiar with kite flying knows some of the difficulties that come with initially sustaining the kite in the air without multiple attempts and without the aid of another person. The problem occurs when the tensile forces in the tether are not adequate to keep the kite aloft. Typically, success comes when the person holding the kite thrusts the kite in the air while the person holing the tether pulls very hard while running. This process can be awkward, requires significant effort, and takes time away from actually flying the kite.

## 2.2. Design Objectives



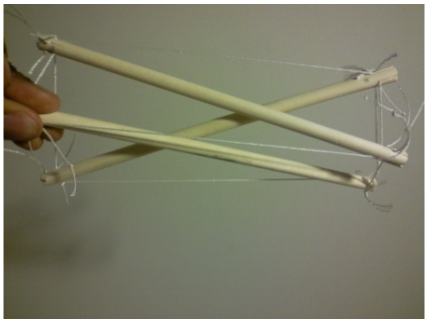
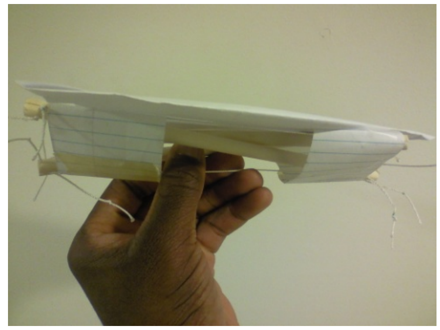
The investigators of this project sought to create a box kite that eliminates the need for another person in order to fly, and simplifies the process of keeping the kite in the air. They will exploit the properties of tensegrity that will allow the collapse of the kite and introduce a launching mechanism that will allow the operator to launch the kite into the air, thus reducing the effort of initially getting the kite to stay suspended in the air.

# 3. CONCEPT DEVELOPMENT

## 3.1. Prototyping

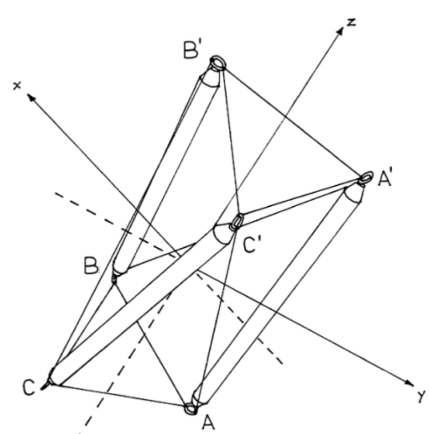
The first attempts at creating the tensegrity kite started from creating basic tensegrity structures using straws and rubber bands. It was from there that the investigators began to explore the different possibilities that would be suitable for the tensegrity kite. The kite needed to be lightweight, comparable in weight to standard kites or less and have similar or lower overall density. This was going to likely be achieved because the number of rigid members was designed to decrease from six. The model had to have the ability to morph, mainly into a configuration that allowed a mechanized launch. Finally the design had to offer an adequate amount of surface area to the air with the intended angle of attack for it to fly. This required a certain amount of symmetry, as well as flat sides like that of a prism.

The investigators began to look at the rod-and-string model for the kite, both for its simplicity, and for its similarity to existing box kite design. The plans for this type of kite included a three-bar triangular prism design with delta wing attachments and possibly a motorized propeller remote control feature.

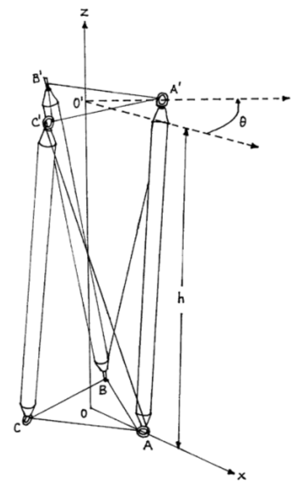
Problems arose with this design, the most notable being with achieving an equilateral triangular prism arrangement. This came from the fact that the arrangement of the rigid members in the equilateral prism shape did not allow for enough tension forces in the tensile members to keep the configuration stable. The idea of using an isosceles triangular prism was considered, but implementing such a design would come at the cost of overall density (lower overall volume) and the asymmetry would complicate the flight mechanics.

The reason the equilateral triangular prism configuration in this case was not possible was the fact that the optimum rotation is fixed at certain angles of one of the equilateral triangular bases. In the book *A Practical Guide to Tensegrity Design,* there is an in-depth analysis of the mathematics that are involved with precisely why that is. The following images and figures come from that book.




The text calls the tensegrity configuration consisting of three solid members arranged somewhat like a triangular prism a "T-prism" where one end is twisted relative to the other base until a tensegrity configuration is achieved. When illustrating this mathematically, a coordinate system is used. The authors of the book rightfully employ a cylindrical coordinate system, first assigning the vertices of the triangular bases and their counterparts, A, B, C, A’, B’, and C’, and then gives them the coordinate as described in the table:

| Point | Coordinates | | |
|---|---|---|---|
| | $z$ | radius | angle |
| $A$ | 0 | $r$ | 0 |
| $B$ | 0 | $r$ | $\frac{2\pi}{3}$ |
| $C$ | 0 | $r$ | $-\frac{2\pi}{3}$ |
| $A'$ | $h$ | $r$ | $\theta$ |
| $B'$ | $h$ | $r$ | $\theta + \frac{2\pi}{3}$ |
| $C'$ | $h$ | $r$ | $\theta - \frac{2\pi}{3}$ |




In this analysis, the lower base, triangle ABC is held stationary, and the upper base, A’B’C’, is rotated through an angle $\theta$. The *z*-coordinate corresponds to the length of the rods when $\theta$ is 0, and $r$ is the planar distance from all of the vertices from the axis of symmetry through the center of the triangles. The variable $t$, used later, will be the length of the chord of our structure, or the wire between a vertex and an adjacent vertex on the opposite base. The 2π/3 comes from the fact that we are working with equilateral triangles. There is only a few rotation angles for which the configuration will be stable. The goal is to fix $r$, so that the triangles remain equilateral, fix $s$, what the book calls the strut length, or the distance between A and A’ (|AA’|) for example, and finally enforce the symmetry on the geometry of the configuration. The variable $t$, used later, will be the length of the chord of our structure, or the wire between a vertex and an adjacent vertex on the opposite base, or the distance from A to C’ (|AC’|) for example. This too will be fixed for our purposes.   Here, the constraints and geometry allow us to keep the geometry desired in our project and allows us to to set up an equation to minimize $\theta$ with a variable $h$ by taking the expression for the chord of a cylinder, substituting the variables we use, and optimizing the chord length using the constraints.

Equation: This is a direct result of our constraints, and is given by the equation of the length of a chord, and derived from the Law of Cosines. This is the value we seek to minimize for stability.

$$t^2 = |AC'|^2 = h^2 + 2r^2 - 2r^2 \cos\left(\frac{2\pi}{3} - \theta\right)$$

Since *s* is constant, we also can assert, because the rigid member length does not change, that:

$$s^2 = |AA'|^2 = h^2 + 2r^2 - 2r^2 \cos(\theta)$$

By combining terms, we can consolidate our equations into one expression with respect to θ.

$$t^2 = s^2 + 2r^2 \cos(\theta) - 2r^2 \cos\left(\frac{2\pi}{3} - \theta\right)$$

By differentiating and solving, we can find the value of θ that satisfies our stability requirements, again, by minimizing t, or the length of the chords that are made up of our tensile members across the vertices:

$$-2r^2 \sin(\theta) - 2r^2 \sin\left(\frac{2\pi}{3} - \theta\right) = 0$$

$$\sin(\theta) = -\sin\left(\frac{2\pi}{3} - \theta\right)$$

$$\sin(\theta) = \sin\left(\theta - \frac{2\pi}{3}\right)$$

Now we know that the sines of the angles θ and (θ-2π/3) can only be equal if their sum is some odd multiple of pi. We disregard their difference because it does not satisfy the equation; it would have to be some even multiple of pi. Here we can force the angles to simply equal to $1\pi$.

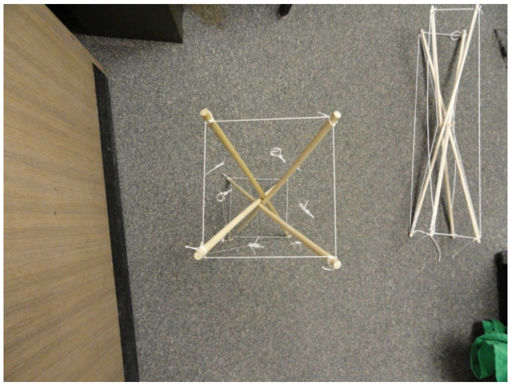
$$\theta + \left(\theta - \frac{2\pi}{3}\right) = \pi$$

The solution to this particular case, where the sum of the angles is between 0 and 2π, is:

$$\theta = \frac{5\pi}{6}$$

Although there are mathematically several potential solutions, there is only one where all of our constraints are satisfied, the structure is stable, and the angles are within reason. Unfortunately, that is not any multiple of 2π/3, and therefore cannot provide us with the true prism shape the investigators sought for this project.

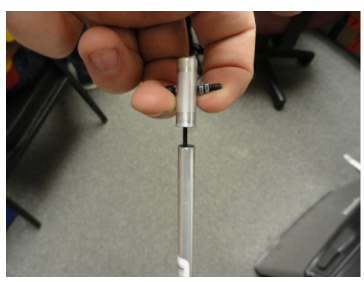
Despite the available literature, the investigators of this project arrived at this conclusion after some trial-and-error. After several attempts at a viable arrangement, the three-bar model was abandoned in favor of the four-bar structure. This model gave rise to several advantages, but at the cost of additional weight. It first allowed for a rectangular prism shape. The design was simpler and more easily duplicated. The similarities between the four-bar tensegrity shape and the original box kite design made the design more comparable and easier to implement. The new arrangement allowed for symmetry within the members, and a higher overall volume.



One key feature that was required for the design was the addition of elastic tensile members along the length of the kite. This solves the problem of having to collapse the kite, making launching feasible and significantly increasing portability.  The investigators also used rings and caps at the end of the rods to preserve the integrity of the rods and to make assembly easier. For the selection materials, the investigators required rods that resisted bending but still lightweight and tensile members that would withstand high tension forces. A force analysis of the ideal tensegrity kite was conducted (located in Appendix III) to indentify the strengths of materials needed. The materials that were selected were carbon fiber for the rods, nylon fiber for the strings at the ends, and nylon bungee cord for the tensile members along the length of the model. Maintaining the rectangular prism structure allowed for the sails to be composed of material that was identical to the original kite. The final structure model and first working prototype is comprised of the aforementioned elements.

## 3.2. Manufacturing and Final Specifications

As mentioned earlier, the components of final structure model were selected based on material properties. After the materials were acquired, the investigators began construction of the final structure model. The model began from the carbon fiber tubes, they were purchased from DragonPlate, a large-scale suppler of carbon fiber materials. The tubes were 3/8" in diameter and 3’ long. The tubes, despite their length, are not easily subjected to bending moments. The caps were constructed from various metals and were slipped over the ends of the tubes where they were held in place with an elastic cord within the rods. The rings, simple keyrings one inch in diameter, were attached to the caps using the holes that were drilled through them. The caps made the rest of the assembly much easier as that it allowed for adjustments. The rods were arranged in such a way each rod had one cap at the corner of the square cross-section at one end and the other cap at the opposite corner of the opposite end, or as if the rods were aligned parallel in a square, then the corners of one end of the "prism" were rotated 180 degrees. Finally the configuration was held in place with ordinary nylon to keep the square shape at the opposite ends and elastic cord along the lengths of the edges for stability, thus completing the model.


### 3.3. Flight Testing

Testing of the first prototype occurred on Friday, March 11, 2011 on Nippert Stadium at the University of Cincinnati in Cincinnati Ohio. There the investigators had a chance to benchmark the kite's performance for additional improvements. The kite's viability was verified, and further work was done on the kite to create a more aesthetically pleasing, and also a more stable solution. The investigators integrated the sails as part of the structure of the kite, instead of affixing them to the outside of the tensile members. The investigators also replaced the sails with a higher-grade material and were pulled to a significantly higher tension to cope with the problems of stability and sustained flight.

The next flight test was conducted at the Cincinnati Kite Festival at Voice of America Park in Mason, Ohio on Sunday April 10, 2011. The test yielded better results than that on Nippert Stadium but improvements still need to be made. During the second flight test, key successes and shortcomings were identified for benchmarking and further refinement. The new configuration of the kite generated a significant amount of force with nominal wind speeds (5-10 mph gusts) and initially getting the kite aloft was took minimal effort, certainly not more than a traditional kite. The issues occurred during the efforts to keep the kite aloft. The kite proved to be almost incapable of sustained flight, quickly generating enough force to lift off the ground, only to return moments later. This especially occurred when the kite drifted clockwise ("right") about the tether point or user. Also, several attempts were made at launching the kite from the "closed" position by the user by throwing the kite, and only once did the kite's sails catch the wind in such a way that the kite took off. The investigators are now charged with the task of exploring potential solutions to the issue of maintaining flight before taking on the task of developing a launching mechanism for deploying the kite with no assistance from other individuals. At this time potential solutions include the introduction of sails to the kite, which would potentially offer more stability during flight by forcing the kite to turn when the wind changes direction or when the kite rotates in the air, and result in longer flight times, and also finding out a way to restrict the rotation of the members when the kite is subjected to the deforming forces of the wind.

## 4. FUTURE RESEARCH

Now that a feasible model has been created, efforts will now go into optimizing the kite, finding and correcting errors, designing the manufacturing processes of the kite itself and begin work on designing the launch mechanism. The ideas currently in place for the launch device involve a tube with a loaded spring that the kite will fold into. The current design for the kite allows it to be completely compressed in such a way that the bars are nearly adjacent and parallel to one another. The problems that the investigators foresee include deciding what type of string to use to gain the desired effect, making sure the kite does not tangle while inside the tube, and safety measures.

## 5. CONCLUSION & REFLECTIONS

There is still work and further research to be done on this project. The major outcomes of the tasks so far include the literature review, idea generation, concept investigation and final concept selection, material selection, and initial testing. The box kite is popular even still because of its simplicity and stability. The intent of this project was to address some of the timeless design's shortcomings, without introducing new ones. The concept of tensegrity was integrated with the current design because of its ability to morph and the ability to economize weight while maintaining structure and stability. The tensegrity design also lends itself to the collapse and mechanical launch of the kite, thus solving the problem of needing another person for launch.

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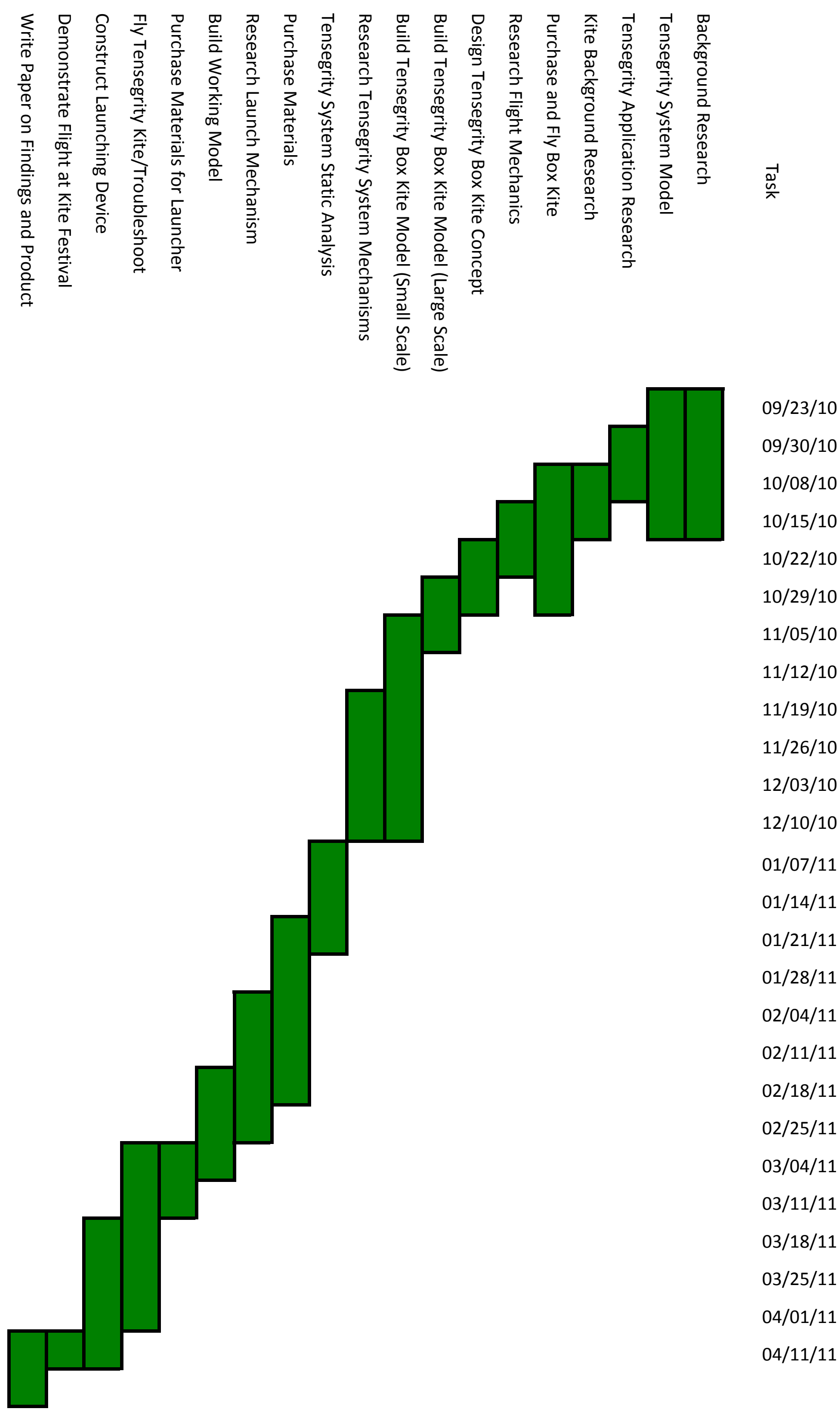
# APPENDIX I: RESEARCH TIMELINE



## APPENDIX II: SAMPLE CALCULATIONS

$l_0 = 7.9\text{cm}$ $\quad m = 179.24\text{ g}$ $\quad l_{rod} = 13.0\text{ cm}$

$l_f = 10.1\text{ cm}$ $\quad F = kx$

$x = (l_f - l_0)\cdot 2$

$0.17924(9.81) = k(0.044)$

$k = 39.962$

* Not accounting for mass of member in compression

Rubber bands in model

$F = kx$

| | | | |
|---|---|---|---|
| $L_1 = 11.0$ cm | $L_2 = 11.0$ cm | $L_3 = 11.0$ cm | $L_4 = 11.5$ cm |
| $x_1 = 6.2$ cm | $x_2 = 6.2$ cm | $x_3 = 6.2$ cm | $x_4 = 6.7$ cm |
| $F_1 = 2.478$ N | $F_2 = 2.478$ N | $F_3 = 2.478$ N | $F_4 = 2.677$ N |

$l_{top} = 22.0$ cm $\quad l_{bot} = 21.0$ cm

$x_{top} = 6.2$ cm $\quad x_{bot} = 5.2$ cm

$F_{top} = 2.478$ N $\quad F_{bot} = 2.078$ N

Face 1